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A Theoretical Analysis of Transient Sound Radiation From a Clamped Circular Plate

A. Akay
Mem. ASME

M. Tokunaga[1]

M. Latcha
Assoc. Mem. ASME

Mechanical Engineering Department,
Wayne State University,
Detroit, Mich. 48202

A theoretical analysis of transient sound radiation from a clamped circular plate is given using a pressure impulse response method. The vibration response of the plate to a transient point force is obtained. The modal pressure impulse response functions for the plate are derived from the Rayleigh surface integral and numerically convoluted with the modal acceleration response of the plate. The impulse response functions are closely related to the mode shapes and the geometry of the problem. They relate the spatial domain to the temporal domain of the pressure waves. The pressure impulse response waveforms are given for a number of plate modes and the changes in the waveforms with distance from the plate are shown. Sound radiation due to forced and free vibrations of the plate are discussed.

Introduction

Acoustic radiation from planar elastic structures is a fundamental problem in acoustics. Transient radiation from vibrating surfaces represents an important subset of this problem in the analysis of a large number of diverse acoustics problems. Earlier work on this subject includes radiation from transient point-excitations of infinite plates [1-3], transient sound transmission through finite plates [4-8], and a large class of problems on transient radiation from a baffled planar piston. An extensive review of the latter subject is given in reference [9].

The specific problem of transient sound radiation from a finite plate excited by an impact has been the subject of a number of studies. However, most of these studies have been experimental in nature and have resulted in empirical relationships between various parameters [10]. A significant analytical study was reported by Strasberg [11] on radiation from a periodically struck diaphragm, where radiated acoustic power was obtained in the frequency domain.

Sound radiation from a vibrating plate in an infinite baffle can be obtained by solving the pressure wave equation with appropriate boundary and initial conditions. A number of mathematical techniques have been developed for the problem of sound radiation from a baffled piston. These can be grouped under four fundamental methods: (1) Rayleigh surface integral [12], (2) King integral [13], (3) Schoch solution [14], and (4) convolution integral [15, 16]. Among these methods only the King integral is limited to circular-shaped pistons. The other three methods are applicable to baffled planar pistons with arbitrary shapes.

The Rayleigh surface integral, which is a special case of the Helmholtz-Huygens Integral, was the first of these methods developed to treat the problem of radiation from baffled planar pistons. The other three methods are solutions of the wave equation; however, they can also be directly derived from the Rayleigh surface integral by appropriate change of coordinates. A detailed discussion of these methods and their interrelationships can be found in reference [9].

The problem of transient radiation from baffled planar radiators with nonuniform vibration amplitude was treated in a number of recent studies [17-20]. Greenspan [17] considered radiation from rigid, simply supported, clamped and Gaussian radiators, treating both steady-state and transient cases. Harris [18] developed a method for transient radiation from baffled planar pistons with arbitrary vibration amplitude. His method is an extension of the convolution method where a generalized spatial impulse response function is convolved with the piston vibration-time history. Harris's results compared the effects of point and finite size receivers. The problems of radiation from planar radiators with axisymmetric vibration distribution was treated by Stepanishen [19]. He developed a generalized Green's function which leads to representation of the acoustic field with a single integral. His results were further generalized for the case of radiators with arbitrary shape and nonuniform vibration distributions by Tjøtta and Tjøtta [20].

The present paper describes a theoretical analysis of transient acoustic radiation from an impact-excited clamped circular plate in an infinite baffle. The expression for the sound pressure is obtained using the spatial impulse response-convolution approach, where the pressure impulse response is

[1] Present Address: Daido-kiko Limited, 4-1-3 Hatchobori, Chuo-ku, Tokyo, Japan 104.

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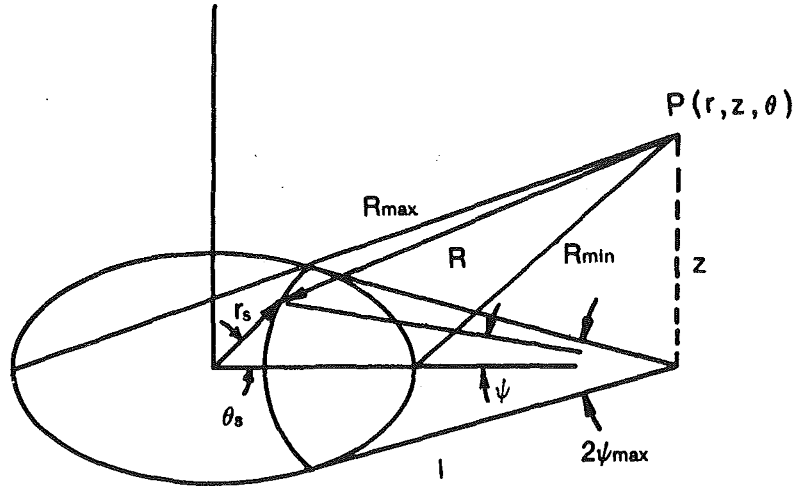

Fig. 1 Geometry of the problem

obtained as a line integral and convolved with the acceleration-time history of the plate to obtain the transient radiation. Several numerical results are given to illustrate the distinct differences in the sound pressure waveforms due to forced and free vibrations of a clamped plate in an infinite baffle excited by an impact force. The results are found to be the same as those obtained by direct numerical integration of the Rayleigh surface integral reported by the authors earlier [21].

Plate Vibrations

Radiation from a thin circular plate of radius **a** clamped at the outer edge and mounted in an infinite baffle is considered. An inviscid isotropic homogeneous fluid with a speed of sound c and a density ρ_0 much less than the density of the plate ρ is in contact with the plate. Geometry of the problem is shown in Fig. 1. The plate is assumed to be excited from rest with a transient point force. For mathematical convenience the Euler plate theory is used to determine the vibratory response of a thin plate. However, as pointed out in reference [3], for frequencies above which the wavelength in the plate is less than eight times its thickness, the Timoshenko-Mindlin plate theory should be used to prevent the slight error introduced into the group and phase velocities.

Using classical plate theory, the equation of motion for the transverse displacement, u, of a plate is given as:

$$D \nabla^4 u + C\partial u/\partial t + \rho h \partial^2 u/\partial t^2 = F(r_s,\theta_s,t) \quad (1)$$

where h is the thickness of the plate and $D = Eh^3/12(1-\nu^2)$ is its flexural rigidity. E is Young's modulus, C is the damping coefficient, and ν is Poisson's ratio. $F(r_s,\theta_s,t)$ is the applied transverse force per unit surface area of the plate.

The general solution to equation (1) for a plate initially undeformed and at rest can be written as:

$$u(r_s,\theta_s,t) = (1/\rho h) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [\phi_{nm}(r_s,\theta_s)/\omega^*_{nm}] \int_0^t F_{nm}(\tau) e^{-\xi\omega_{nm}(t-\tau)} \sin \omega^*_{nm}(t-\tau) d\tau \quad (2)$$

where $\omega^*_{nm} = \omega_{nm}\sqrt{1-\xi^2}$. ξ is the damping ratio and ω_{nm} are the natural frequencies of the plate, with the subscripts n and m denoting the radial and circular modes, respectively. ϕ_{nm} are the normal modes of the plate and are normalized using their orthogonality property as:

$$\int_0^{2\pi}\int_0^a \phi_{nm}(r_s,\theta_s)\phi_{pq}(r_s,\theta_s) r_s dr_s d\theta_s = \pi a^2 \delta^{pq}_{nm} \quad (3)$$

where

$$\delta^{pq}_{nm} = \begin{cases} 1 & n=p,\ m=q \\ 0 & n\neq p,\ m\neq q \end{cases}$$

The modal coefficients of the force $F_{nm}(t)$ are found as

$$F_{nm}(t) = \left[\int_0^{2\pi}\int_0^a F(r_s,\theta_s,t)\phi_{nm}(r_s,\theta_s) r_s dr_s d\theta_s\right] \Big/ \left[\int_0^{2\pi}\int_0^a \phi^2_{nm}(r_s,\theta_s) r_s dr_s d\theta_s\right]$$

A transient point force may be expressed as

$$F(r_s,\theta_s,t) = F_0 \delta(r_s - r_0)\delta(\theta_s - \theta_0) f(t)/2\pi r_s \quad (5)$$

where $f(t)$ is the arbitrary time dependence of the force and δ is the Dirac delta function. When the transient force is applied at the center of the plate, the force function takes the form

Nomenclature

- **a** = radius of plate
- c = wave speed in acoustic medium
- h = thickness of plate
- r,z,θ = cylindrical coordinates
- r_s,θ_s = polar coordinates on the plate surface
- u = transverse displacement of plate
- C = damping coefficient
- D = flexural rigidity of plate $= Eh^3/12(1-\nu^2)$
- E = Young's modulus
- F = applied load
- M = mass of plate
- $\delta(r)$ = Dirac delta function
- λ_n = freqency parameter
- ξ = damping ratio
- ν = Poisson's ratio for plate
- ρ = density of plate material
- ρ_0 = density of acoustic medium
- ϕ_n = eigenfunctions of plate nth mode
- ω_n = natural frequency of a clamped circular plate
- ω_0 = π/duration of transient force
- ω^*_n = $\omega_n\sqrt{1-\xi^2}$

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$$F(r_s,t)=F_0\delta(r_s)f(t)/2\pi r_s \tag{6}$$

and the axisymmetric response of the plate is written as

$$u(r_s,t)=\frac{1}{\rho h}\sum_{n=0}^{\infty}\frac{\phi_n(r_s)}{\omega_n^*}\int_0^t F_n(\tau)\sin\omega_n^*(t-\tau)\,d\tau \tag{7}$$

where

$$F_n(t)=\int_0^{2\pi}\int_0^a F(r_s,t)\phi_n(r_s)r_s dr_s d\theta_s\Big/\int_0^{2\pi}\int_0^a \phi_n^2(r_s)r_s dr_s d\theta_s \tag{8}$$

The mode shapes ϕ_n and the natural frequencies ω_n are determined by applying the boundary conditions to the homogeneous undamped equation of motion of the plate. For a circular plate with a clamped edge, the axisymmetric mode shapes and the natural frequencies are given by [22]:

$$\phi_n(r_s)=\frac{1}{\sqrt{2}}\left[\frac{J_0(\lambda_n r_s/a)}{J_0(\lambda_n)}-\frac{I_0(\lambda_n r_s/a)}{I_0(\lambda_n)}\right] \tag{9}$$

$$\omega_n=(\lambda_n/a)^2\sqrt{D/\rho h} \tag{10}$$

where λ_n are the roots of the frequency equation (11):

$$J_0(\lambda_n)I_1(\lambda_n)+J_1(\lambda_n)I_0(\lambda_n)=0. \tag{11}$$

J_0 and J_1 are Bessel functions of the first kind and I_0 and I_1 are modified Bessel functions of the first kind.

The transient force-time history is represented with a squared half-sine pulse in equation (6) to simulate elastic impact of a plate with a sphere [21]:

$$F(r_s,t)=\begin{cases} F_0\delta(r_s)\sin^2\omega_0 t/2\pi r_s, & 0\le t\le \pi/\omega_0 \\ 0 & , \quad \pi/\omega_0\le t\end{cases} \tag{12}$$

The axisymmetric displacement response in equation (7) to the forcing function in (12) becomes:

$$u(r_s,t)=\frac{F_0}{M}\sum_{n=0}^{\infty}\frac{\phi_n(0)\phi_n(r_s)}{(4\omega_0^2-\omega_n^2)\sin\Omega-4\xi\omega_n\omega_0\cos\Omega}$$

$$\begin{cases} \tfrac{1}{2}\sin(2\omega_0 t+\Omega)+X_1\exp(-\xi\omega_n t)\sin(\omega_n^* t_1+\Omega_1)+E, \\ \qquad 0\leqq t\leqq \pi/\omega_0 \\ X_2\exp(-\xi\omega_n t_1)\sin(\omega_n^* t_1+\Omega_2), \quad t_1\geqq 0 \end{cases} \tag{13}$$

where

$\Omega = \tan^{-1}[(\omega_n^2-4\omega^2)/4\xi\omega_0\omega_n]$,

$\Omega_1 = \tan^{-1}\{\omega_n^*(E+\tfrac{1}{2}\sin\Omega)/[\omega_0\cos\Omega+\xi\omega_n(E+\tfrac{1}{2}\sin\Omega)]\}$

$\Omega_2 = \tan^{-1}[C_1\omega_n^*/(C_2+C_1\xi\omega_n)]$,

$X_1 = [\omega_0\cos\Omega/(\xi\omega_n\sin\Omega_1-\omega_n^*\cos\Omega_1)]$

$X_2 = [C_2/(\omega_n^*\cos\Omega_2-\xi\omega_n\sin\Omega_2)]$,

$C_1 = \tfrac{1}{2}\sin\Omega+X_1\exp(-\xi\omega_n\pi/\omega_0)\sin[(\omega_n^*\pi/\omega_0)+\Omega_1]+E$

$C_2 = \omega_0\cos\Omega+X_1\exp(-\xi\omega_n\pi/\omega_0)\{\omega_n^*\cos[(\omega_n^*\pi/\omega_0)+\Omega_1)]-\xi\omega_n\sin[(\omega_n^*\pi/\omega_0)+\Omega_1]\}$

$E = [(4\omega_0^2-\omega_n^2)\sin\Omega-4\xi\omega_0\omega_n\cos\Omega]/2\omega_n^2$,

$M = \rho\pi a^2 h, t_1=t-\pi/\omega_0, \omega_n^*=\omega_n\sqrt{1-\xi^2}$

Acoustic Radiation

The sound pressure radiated from a vibrating plate in a rigid infinite baffle ($z=0$ plane) can be obtained by evaluating the Rayleigh surface integral [12]. For an axisymmetric radiator the Rayleigh integral is written as:

$$p(r,z,t)=(\rho_0/2\pi)\int_S \ddot{u}[r_s(l,\psi),t-R/c]dS/R \tag{14}$$

where $\ddot{u}(r_s,t)$ is the acceleration-time history of plate vibrations and R is the distance from the receiver to any point on the plate surface. The elemental surface area is $dS = ldld\psi$ with the coordinates l,ψ shown in Fig. 1. Subscript s delineates the coordinates on the plate surface.

In the Rayleigh integral the delay in the arrival of the acoustic pressure waves emanating from points on the plate at different distances from the receiver point is taken into account by the delay-time ($t-R/c$) in the acceleration term. The integration is carried out over the plate surface.

The spatial impulse response or the convolution approach can be developed directly from the Rayleigh integral in equation (14) by first transforming the integration variables from the surface coordinates (r_s,θ_s) to receiver-oriented coordinates (R,ψ) and then performing one of the integrals. Referring to Fig. 1 and considering the relationship between the coordinates, $R^2 = z^2 + l^2$ and $RdR = ldl$, equation (14) can be written as

$$p(r,z,t)=\frac{\rho_0}{2\pi}\int_{R_{\min}}^{R_{\max}}\int_0^{2\psi_{\max}(R,z)}\ddot{u}[r_s(R,z,\psi),t-R/c]dRd\psi \tag{15}$$

where $R_{\min}$ and $R_{\max}$ are the closest and farthest distances from the receiver to the plate surface. $2\psi_{\max}$ is the largest angle ψ makes as shown in Fig. 1.

From equation (13) the axisymmetric acceleration response of an impact-excited clamped circular plate can be written as

$$\ddot{u}(r_s,t)=\sum_{n=1}^{\infty}\phi_n(r_s)\ddot{q}_n(t) \tag{16}$$

where

$$\ddot{q}_n(t)=\left(\frac{F_0}{M}\right)\frac{\phi_n(0)}{(4\omega_0^2-\omega_n^2)\sin\Omega-4\xi\omega_n\omega_0\cos\Omega}$$

$$X\begin{cases} -2\omega_0^2\sin(2\omega_0 t+\Omega)+X_1\exp(-\xi\omega_n t)[(\xi^2\omega_n^2-\omega_n^{*2}) \\ \qquad \sin(\omega_n^* t_1+\Omega_1)-2\xi\omega_n\omega_n^*\cos(\omega_n^* t+\Omega_1)], \\ \qquad 0\leqq t\leqq \pi/\omega_0 \\ X_2\exp(-\xi\omega_n t_1)[(\xi^2\omega_n^2-\omega_n^{*2})\sin(\omega_n^* t_1+\Omega_2) \\ \qquad -2\xi\omega_n\omega_n^*\cos(\omega_n^* t_1+\Omega_2)], \quad t_1\geqq 0 \end{cases}$$

$\phi_n(r)$ is given in equation (9).

Substitution of equation (16) in (15) gives:

$$p(r,z,t)=\frac{\rho_0}{2\pi}\int_{R_{\min}}^{R_{\max}}\int_0^{2\psi_{\max}(R,z)}\left\{\sum_{n=1}^{\infty}\phi_n[r_s(R,z,\psi)]\ddot{q}_n(t-R/c)\right\}dRd\psi \tag{17}$$

After changing the order of summation and integrations, and substituting $\tau=R/c$, equation (17) can be written as:

$$p(r,z,t)=\sum_{n=1}^{\infty}\int_{\tau_{\min}}^{\tau_{\max}}\ddot{q}_n(t-\tau)\left\{\frac{\rho_0 c}{\pi}\int_0^{\psi_{\max}(c\tau,z)}\phi_n[r_s(R,z,\psi)]_{R=c\tau}d\psi\right\}d\tau \tag{18}$$

Equation (18) shows the sound pressure at the receiver point to be a result of radiation at each mode, which can be written as:

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$$p(r,z,t)=\sum_{n=1}^{\infty}[\ddot{q}_n(t)^* p_{in}(r,z,t)]$$

$$R_{\min}/c \leqq t \leqq R_{\max}/c \qquad (19)$$

where the modal impulse response function at the nth mode, p_{in}, is given as:

$$p_{in}(r,z,t)=\frac{\rho_0 c}{\pi}\int_0^{\psi_{\max}(R=ct,z)}\phi_n[r_s(R,z,\psi)]_{R=ct}\,d\psi \qquad (20)$$

Equations (18) and (19) can be interpreted as convolution of plate acceleration-time history with the impulse response function, $p_{in}(t)$, at each mode. These results can be obtained using either the convolution integral method [9] or the impulse response approach developed using a Green's function solution [19].

Substituting equations (24) in equation (20) and performing the integration results in the following modal pressure impulse response expressions:

I. When $r > a$:

$$p_{in}(r,z,t)=\frac{\rho_0 c}{\pi\sqrt{2}}\left\{\sum_{m=0}^{\infty}\epsilon_m J_m(\lambda_n r/a)J_m[\lambda_n(c^2t^2-z^2)^{1/2}/a]\right.$$

$$\left./J_0(\lambda_n)-\sum_{m=0}^{\infty}\epsilon_m(-1)^m I_m(\lambda_n r/a)I_m[\lambda_n(c^2t^2-z^2)^{1/2}/a]/I_0(\lambda_n)\right\}$$

$$\frac{\sin m\psi}{m},\quad R_{\min}\leqq ct\leqq R_{\max} \qquad (25)$$

II. When $r \leqq a$:

$$p_{in}(r,z,t)=\begin{cases}\dfrac{\rho_0 c}{\sqrt{2}}\{[J_0(\lambda_n r/a)J_0[\lambda_n(c^2t^2-z^2)^{1/2}/a]/J_0(\lambda_n)] \\ -[I_0(\lambda_n r/a)I_0[\lambda_n(c^2t^2-z^2)^{1/2}/a]/I_0(\lambda_n)]\},\quad z\leqq ct\leqq R_{\min} \\ \text{same as equation (25)},\quad R_{\min}\leqq ct\leqq R_{\max}\end{cases} \qquad (26a)$$

It should be noted that the modal impulse response function, $p_{in}(t)$, relates the radiated acoustic pressure to the receiver-source geometry and to the mode shapes of the radiator. Further discussion on modal impulse response will be given later in this paper.

Evaluation of the Modal Impulse Response

The modal impulse response function, given in equation (20), is evaluated by first expressing the mode shape functions $\phi_n(r_s)$ in terms of the receiver coordinates as $\phi_n(R,\psi)$. Using the law of cosines and referring to Fig. 1:

$$r_s^2=r^2+R^2-z^2-2r(R^2-z^2)^{1/2}\cos\psi \qquad (21)$$

The values of the angle ψ are defined as:

I. When $r>a$:

$$\psi(R,z)=\cos^{-1}[(R^2-z^2+r^2-a^2)/2r(R^2-z^2)^{1/2}],$$
$$R_{\min}\leqq ct\leqq R_{\max} \qquad (22)$$

II. When $r\leqq a$:

$$\psi(R,z)=\begin{cases}\pi, & z<ct\leqq R_{\min}\\ \cos^{-1}[(R^2-z^2+r^2-a^2)/2r(R^2-z^2)^{1/2}], & R_{\min}\leqq ct\leqq R_{\max}\end{cases} \qquad (23)$$

where $R_{\min}=[z^2+(r-a)^2]^{1/2}$ and $R_{\max}=[z^2+(r+a)^2]^{1/2}$.

The integration of the Bessel functions with respect to ψ can be simplified by using Neumann's addition theorem [23,24]:

$$J_0(\lambda_n r_s/a)=J_0(\lambda_n r/a)J_0[\lambda_n(R^2-z^2)^{1/2}/a]$$
$$+2\sum_{m=1}^{\infty}J_m(\lambda_n r/a)J_m[\lambda_n(R^2-z^2)^{1/2}/a]\cos m\psi \qquad (24a)$$

A similar relationship can be obtained for the modified Bessel functions as:

$$I_0(\lambda_n r_s/a)=I_0(\lambda_n r/a)I_0[\lambda_n(R^2-z^2)^{1/2}/a]$$
$$+2\sum_{m=1}^{\infty}(-1)^m I_m(\lambda_n r/a)I_m[\lambda_n(R^2-z^2)^{1/2}/a]\cos m\psi \qquad (24b)$$

where $\epsilon_0 = 1$ and $\epsilon_m = 2$ for $m \geqq 1$.

The first part of equation (26*a*) is similar to the result for a membrane given in [19]. Since the clamped plate cannot have rigid body motion, the result for a rigid piston, obtained by substituting $\lambda_n = 0$ in equation (26*a*), indicates zero sound pressure.

The acoustic radiation from a clamped plate excited by a transient force such as that given in equation (12) can be found by substituting equations (25) and (26) in the convolution integral given in equation (19).

Results

The transient sound pressure radiated from a clamped plate is obtained by evaluating the convolution for each mode and summing the results numerically. For the numerical examples, the case of radiation from a 0.50 m diameter steel plate of 1.59 mm thickness was used. The magnitude of the excitation force in 134 N with the force-time history given in equation (12).

The plate acceleration response at its midpoint, the pressure impulse response at $z = 0.10$ m on the axis of symmetry, and the resulting modal sound pressure at the same location are shown in Fig. 2 for a number of modes. The total plate acceleration response and sound pressure waveforms are obtained by summing the modal acceleration and sound pressure waveforms for 50 modes, as shown in Fig. 3. The sound pressure waveform in Fig. 3 shows a distinct pressure pulse before the resonant radiation or "ringing" starts. The initial pressure pulse is due to the forced deformation of the plate where the pressure waveform duplicates the plate velocity waveform at the excitation point. There is no sound radiation until the plate bending waves reflect from the plate edge back to the plate center. Since bending waves above the critical frequency radiate toward the direction of propagation, on the axis of symmetry sound pressure from the plate resonant vibrations is realized only after the bending waves reflect from the edge of the plate [21].

The pressure impulse response function is shown in Fig. 4 for the tenth mode at various distances from the center of the plate on its axis of symmetry. It is seen that as the receiver point moves away from the plate surface, the pressure waveform is compressed in time. This time compression is

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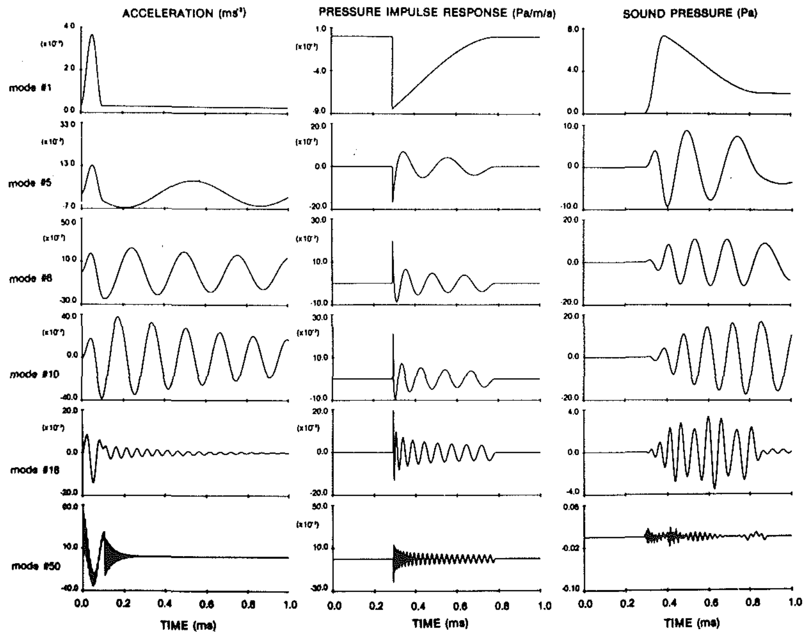

Fig. 2 The modal acceleration response, pressure impulse respone, and corresponding sound pressure radiation for various modes of the plate. $z = 0.10$ **m,** $\xi = 0.025$**.**

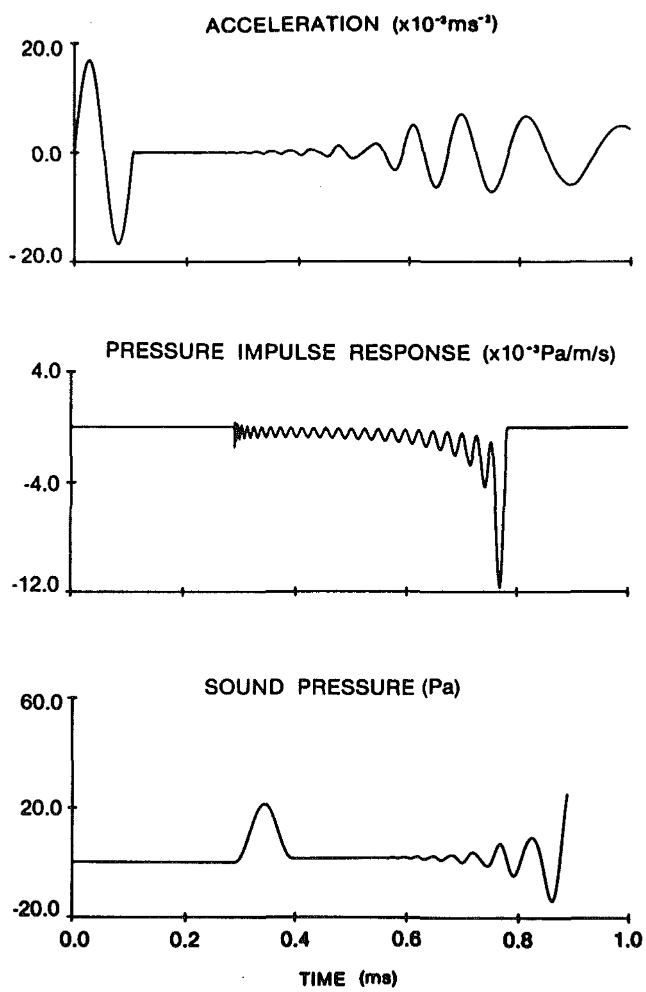

Fig. 3 The plate acceleration, pressure impulse response, and sound pressure obtained using the first 50 modes of the plate. $z = 0.10$ **m,** $\xi =$ **0.025.**

pronounced at the start of the waveform. This, of course, is due to the decrease in the difference between the time of arrival of sound waves from various points on the plate surface. Mathematically, it can be seen in the pressure impulse response equations (25) and (26), where the form of the argument, $(c^2 t^2 - z^2)^{1/2}$, leads to the time compression of the waveform. Another implication of this time compression is the inherent change of the frequency content of the pressure impulse response with distance. The sound pressure spectrum is shifted into higher frequencies as the distance from the plate is increased. The transition in the spectrum continues until $z \cong R_{\min}$ in equation (26*a*). When the approximation $z \cong R_{\min}$ is true, the receiver point is said to be in the acoustic far field of the plate. In the far field the spectral content is stable. However, when the receiver point is near the plate, where $z \cong R_{\min}$ is no longer true, lower frequency sound field is apparent in the pressure impulse response waveforms as shown in Fig. 4.

It should be noted that the changes in the pressure impulse waveform and its spectrum with distance are strictly due to the mode shapes and the geometry of the problem. The physical reasoning behind these changes is the interference of the pressure waves from different parts of the plate leading to cancellation at each mode. The results of the convolutions are modified accordingly. Although not shown here, similar time compression effects are also seen in the total sound pressure waveforms.

Summary and Conclusions

A sound pressure impulse response function is developed from the Rayleigh integral for radiation from a clamped plate in an infinite baffle. The time-dependent sound radiation from transient excitation of the plate is then found by a convolution of the plate acceleration waveform with the pressure impulse response. The results of this method are the same as those obtained by direct integration of the Rayleigh integral and experimental results reported earlier [21]. The time-dependent boundary conditions that are necessary for transient radiation when using the Rayleigh integral are in the

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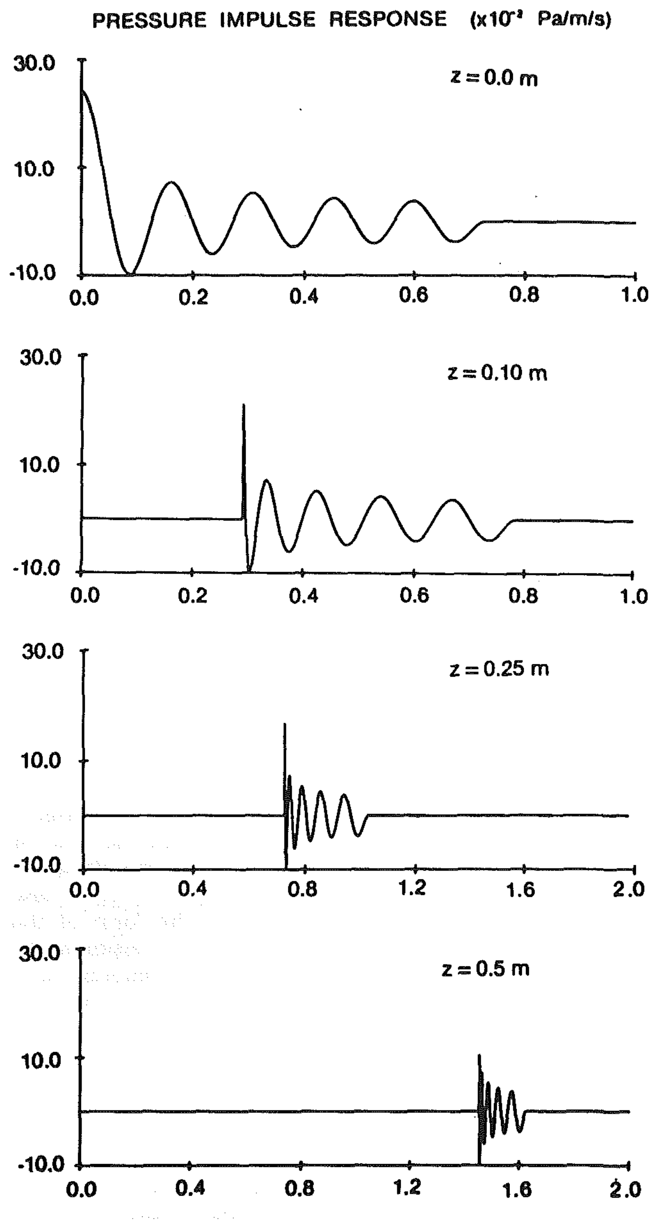

Fig. 4 The tenth modal pressure impulse response at various distances from the plate surface. ξ = 0.025.

form of time-dependent integrand and upper integration limit of the convolution integral in the present method.

An examination of the pressure impulse response in equations (25) and (26) reveals its close relationship to the vibration mode shapes and hence to the geometry and the boundary conditions of the plate. Hence, the pressure impulse response function developed here is a spatial phenomenon affecting the temporal domain. The resulting sound pressure radiation is also directly related to the acceleration response of the plate vibrations. The decay in the time history of the plate vibrations and the resulting radiation is a result of the damping of the plate material. In addition to the rate of decay, the damping of the plate material determines the pressure amplitude, particularly if the plate is excited at one of its resonances.

In the present problem formulation the plate vibration is uncoupled from the acoustic radiation, neglecting the fluid-loading effects on the plate. In general, the influence of fluid loading on plate vibrations is to shift the resonant frequencies and alter the mode shapes. The magnitude of the change depends on the frequency and on the nondimensional parameter $\beta = \rho_0 a/\rho h$, where a,h are the plate radius and thickness and ρ_0, ρ are the density of the fluid and the plate material respectively. For values of $\beta<1$, the effect of fluid loading on the plate due to acoustic radiation is negligible, which is the case for radiation into air from metal plates [25].

The time history of the excitation force applied in the present problem is a smooth pulse without any discontinuities in its slope leading to a smooth acceleration response of the plate. A simple analysis of equation (16) shows that as the duration of the pulse, π/ω_0, is increased, the amplitude of the plate acceleration and therefore the radiation pressure amplitude decreases while shifting the spectrum into low frequencies. For the limiting case of an infinitely long pulse duration ($\omega_0 \rightarrow 0$), acoustic pressure amplitude vanishes. On the other hand, as the duration of the excitation force decreases, the radiation spectrum shifts into higher frequencies by virtue of the resonance.

The acoustic radiation from the plate shows characteristics similar to that from a rigid piston; however, in the case of a transiently excited large plate, radiation from the center of the plate reaches the receiver before the waves from the edge arrive. In addition, the amplitude of the vibrations near the restrained edge is usually small compared to the center of the plate where excitation takes place. Therefore, during the initial part of the transient radiation from a point-excited plate, the pressure on the axis of symmetry is dominated by the sound waves induced by the forced vibration of the plate, duplicating the plate velocity response. It is then followed by the resonant radiation from the plate.

The method developed here is applicable to axisymmetric vibrations of circular plates, membranes, and annular rings with various boundary conditions.

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